DEVELOPMENT OF SECONDARY FREE-CONVECTION CURRENTS IN FORCED TURBULENT FLOW IN HORIZONTAL TUBES

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The problem of the development of secondary free-convection currents in forced turbulent flow in horizontal tubes for relatively weak thermal gravitation influence is analytically solved. The results of the solution are compared to experimental data.

Experimental data on local heat transfer [1] and on velocity and temperature profiles [2, 3] demonstrate that thermal gravitational forces exert a substantial influence on turbulent flow and heat exchange in horizontal tubes. Thermogravitational forces can affect the structure of the turbulence, which results in a variation in momentum and heat transfer and directly affects the averaged flow, which leads to the formation of secondary free-convection currents (as is the case in viscous-gravitational flow). Secondary free-convection currents for a turbulent flow may substantially differ from the pattern of secondary currents for viscous-gravitational flow in horizontal tubes due to high anisotropy and inhomogeneity of the momentum and heat transfer.

The boundaries and nature of the onset of the influence of thermogravitational forces on turbulent momentum and heat transfer have been examined [4] assuming that they do not directly influence the averaged flow. The threshold for the influence of thermogravitational forces on the velocity, temperature, frictional drag, and heat-transfer fields were clarified. The formation of secondary flows was not discussed in this article.

In this work the development of secondary free-convection flows in forced turbulent motion of an incompressible liquid in horizontal tubes will be discussed. The problem is solved by assuming that thermogravitational forces do not affect turbulent transfer. Conditions will be examined for a weak influence of thermogravitational forces, i.e., at relatively low Grashof numbers $Gr = g\beta q_W d^4/\lambda \nu^2$.

By stating the problem this way we are able to clarify the contribution of mass forces for averaged motion and to approximately describe the flow at relatively low Gr.

We will write the kinetic equation for the eddy component as follows:

$$u\frac{\partial\omega_x}{\partial x}+v\frac{\partial\omega_x}{\partial r}+\frac{w}{r}\frac{\partial\omega_x}{\partial\varphi}+\frac{\partial r\overline{v'\omega_x'}}{r\,\partial r}+\frac{1}{r}\frac{\partial\overline{w'\omega_x'}}{\partial\varphi}=\omega_x\frac{\partial u}{\partial x}+ \tag{1}$$

$$+\omega_r\frac{\partial u}{\partial r}+\frac{\omega_\varphi}{r}\frac{\partial u}{\partial\varphi}+\frac{\partial r\overline{u'\omega_r'}}{r\,dr}+\frac{\partial\overline{u'\omega_\varphi'}}{r\,\partial\varphi}+\nu\Delta\omega_x+\frac{g\beta}{r}\left[\frac{\partial(rt)}{\partial r}\sin\varphi+\frac{\partial t}{\partial\varphi}\cos\varphi\right]$$

$$\omega_x=\frac{1}{r}\left[\frac{\partial}{\partial r}(rw)-\frac{\partial v}{\partial\varphi}\right]$$

$$\omega_r=\frac{1}{r}\left[\frac{\partial u}{\partial\varphi}-\frac{\partial}{\partial x}(rw)\right],\quad \omega_\varphi=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial r}$$

where r is the current radius; x is the axial coordinate, counted off from the onset of heating; φ is an angle measured from the upper generatrix; t is temperature; u, v, and w are the axial, radial, and tangential components of the velocity, respectively; ν is the kinematic viscosity coefficient; and β is the thermal-expansion coefficient.

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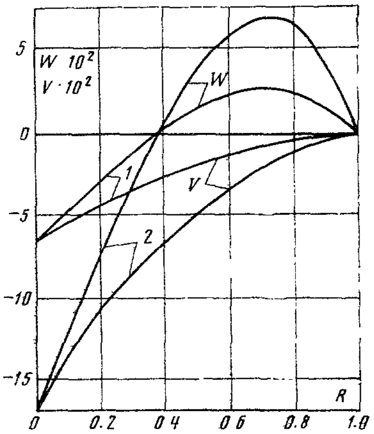

Fig. 1

The problem will be solved under the following assumptions. 1. The process is steady-state. 2. The physical properties of the liquid are constant except for a variation in density that can be taken into account in the mass force term. 3. Flow is stabilized, i.e., the variation of all the hydrodynamic variables in the longitudinal coordinate are negligibly small. 4. Molecular transfer is negligibly small in comparison with turbulent transfer. 5. Turbulent vorticity transfer is represented in a gradient form, i.e., $\overline{v'\omega_x'} = -\varepsilon\partial\omega_x/\partial r$. 6. The turbulent vorticity transfer coefficient is equal to the turbulent momentum transfer coefficient and is described by Prandtl's dependence $\varepsilon/\nu = 0.4\eta$. 7. Prandtl's turbulence number $Pr_T = 1.9$. The heat flux density on the wall is constant and the flow region beyond the onset of the heated section where $\partial t/\partial x = \text{const}$ is considered.

Equation (1) in linearized dimensionless form, taking into account the above assumptions, takes the form

$$\frac{\partial}{\partial R}\left(RY\frac{\partial \Omega_x}{\partial R}\right) = \Gamma\frac{\partial (RT_0{}^+)}{\partial R}\sin\varphi \tag{2}$$

where $\Gamma = 1.25\ \mathrm{Gr/ReRe_*^2Pr}$ is a small parameter,

$$\Omega_x = \frac{\omega_x d}{2\bar{u}} = \frac{1}{R}\left[\frac{\partial}{\partial R}(RW) - \frac{\partial V}{\partial \varphi}\right] \tag{3}$$

$T_0{}^+ = (t_W - t)_0\rho c_p v_*/q_W$ is the initial temperature distribution characteristic for forced flow without the influence of thermogravitation; $R = 2r/d = 1 - Y$ is the current dimensionless radius; d is the tube diameter; $v_* = \sqrt{\tau_W/\rho}$ is the velocity of friction; $V = v/\bar{u}$ and $W = w/\bar{u}$; $\bar{u}$ is the mean rate of flow; q_W is the heat flux density on the wall; $\mathrm{Re} = \bar{u}d/\nu$, $\mathrm{Re}_* = v_*d/\nu$ is the Reynolds number, Pr is Prandtl's number; ρ is density; and c_p is specific heat.

Suppose the temperature profile $T_0{}^+$ is given by

$$T_0{}^+ = 2.2\ln\eta \times \mathrm{B(Pr)} \tag{4}$$

where B(Pr) is the Prandtl number function described according to data [5] by the expression

$$\mathrm{B} = 5\ln((5\,\mathrm{Pr} + 1)/30) + 8.55 + 5\mathrm{Pr};$$

$\eta = v_*y/\nu$ is a dimensionless coordinate counted off from the wall.

Solving Eqs. (2) and (4), we find an expression that describes the distribution of the eddy component:

$$\Omega_x = -\Gamma\{1.1(\ln Y)^2 + \mathrm{D}\ln Y + C_1\}\sin\varphi \tag{5}$$
$$\mathrm{D} = 2.2\ln\eta_a + \mathrm{B(Pr)} = 4.5\lg\mathrm{Re} + \mathrm{B(Pr)} - 5,\quad \eta_a = v_*d/2\nu$$

The boundary condition has the form

$$\left.\begin{matrix}\dfrac{d\Omega_x}{\partial R} = 0, & R\to 0\\ & \varphi = \pi/2\end{matrix}\right\}$$

Using the definition of Ω_x from Eq. (4) and the continuity equation

$$\partial(RV)/\partial R + \partial W/\partial\varphi = 0 \tag{6}$$

we write down an equation for determining the tangential component of the velocity W:

$$\frac{\partial}{\partial R}R\frac{\partial}{\partial R}(RW) + \frac{\partial^2 W}{\partial\varphi^2} = \frac{\partial}{\partial R}(R^2\Omega_x) \tag{7}$$

Representing the desired function W in the form of a product of two functions,

$$W = F(R)\sin\varphi \tag{8}$$

the equation in partial derivatives of Eq. (7) is transformed into an ordinary differential equation,

$$\frac{d^2(RF)}{dR^2} + \frac{d(RF)}{R\,dR} - \frac{(RF)}{R^2} = \frac{1}{R}\frac{d}{dR}(R^2A) \tag{9}$$

where $A = \Omega_x/\sin\varphi$.

The general solution of Eq. (9), according to [6], can be represented in the form

$$F=\frac{1}{2}\left\{\int A\,dR+\frac{1}{R^2}\int R^2 A\,dR+C_2+C_3\frac{1}{R^2}\right\} \tag{10}$$

Substituting the expression for $A=\Omega_x/\sin\varphi$ from Eq. (5) in Eq. (10) and integrating, we find an expression for the component of velocity W:

$$W=F\sin\varphi=\Gamma\sum_{n=1}^{\infty}\frac{n-2(n+1)R+(n+2)R^{n+1}}{n(n+1)(n+3)}\left[D-2.2\sum_{k=1}^{n-1}\frac{1}{k}\right]\sin\varphi \tag{11}$$

where the decomposition

$$\ln(1-R)=-\sum_{n=1}^{\infty}R^n/n$$

is used in integrating.

To find the constants C_1, C_2, and C_3 we use the following conditions: 1) $\int_0^1 F\,dR=0$; 2) $F|_{R=1}=0$; 3) when $R=0$, F is finite.

Using Eq. (11), the continuity equation (6), and the boundary condition $V|_{R=1}=0$, we find an expression for the radial component of the velocity,

$$V=-\Gamma\sum_{n=1}^{\infty}\frac{n-(n+1)R+R^{n+1}}{n(n+1)(n+3)}\left[D-2.2\sum_{k}^{n-1}\frac{1}{k}\right]\cos\varphi \tag{12}$$

Figure 1 depicts the distribution of the tangential component of velocity W in the horizontal center-line plane and the radial component of velocity V in the vertical center-line plane calculated for $\Pr=0.7$ (curves 1) and $\Pr=3.5$ (curves 2) and $\mathrm{Re}=10^4$ and $\mathrm{Re}=5\cdot10^4$ when $\Gamma=10^{-2}$. The difference of the curves for $\mathrm{Re}=10^4$ and $\mathrm{Re}=5\cdot10^4$ is insignificant, so that lines are drawn in the figure constructed for the mean values for the given interval of Re numbers. When $R=0$, the derivatives $\partial V/\partial R$ and $\partial W/\partial R$ are finite. This is due to the use of Eq. (3) in the calculation which does not satisfy $\partial t/\partial r|_{r=0}=0$.

The distribution of the axial-velocity component u and of temperature will be found from the equations

$$\frac{1}{r}\frac{\partial}{\partial r}r\varepsilon\frac{\partial u}{\partial r}=v\frac{\partial u_0}{\partial r}+\frac{1}{\rho}\frac{\partial P}{\partial x} \tag{13}$$

$$\frac{1}{r}\frac{\partial}{\partial r}r\varepsilon\frac{\partial t}{\partial r}=v\frac{\partial t_0}{\partial r}+u_0\frac{\partial t_0}{\partial x} \tag{14}$$

in which convection terms (the first terms in the right sides of the equations) are written under the assumption that $u=u_0$ and $t=t_0$, i.e., they correspond to distributions without the influence of thermogravitation.

We obtain by solving Eqs. (13) and (14) expressions for dimensionless velocity and temperature

$$U^+=U_0^+-\int_0^{\eta}\left[\int_0^{\eta}(\eta_a-\eta)V^+(\partial U_0^+/\partial\eta)\,d\eta\right]((\eta_a-\eta)\varepsilon/\nu)^{-1}\,d\eta \tag{15}$$

$$T^+=T_0^+-\int_0^{\eta}\left[\int_0^{\eta}(\eta_a-\eta)V^+(\partial T_0^+/\partial\eta)\,d\eta\right][(\eta_a-\eta)\varepsilon/\nu]^{-1}\,d\eta \tag{16}$$

$$U^+=u/v_*,\quad V^+=v/v_*,\quad T^+=(t_w-t)\rho c_p v_*/q_w$$

Equation (12) for the radial-velocity component is too awkward to use for solving Eqs. (15) and (16). The curves calculated using Eq. (12) and depicted in Fig. 1 are therefore approximated by the expression

$$V=-7.8\Gamma\sqrt{\Pr}\,Y^2\cos\varphi \tag{17}$$

Using Eqs. (4) and (17), we obtain a dependence that describes the profile of the axial component of velocity and temperature,

$$U^+=U_0^++2\cdot10^5\frac{\mathrm{Gr}}{\mathrm{Re}^{4.37}\sqrt{\Pr}}\eta^2\cos\varphi \tag{18}$$

$$T^+=T_0^++1.8\cdot10^5\frac{\mathrm{Gr}}{\mathrm{Re}^{4.37}\sqrt{\Pr}}\eta^2\cos\varphi \tag{19}$$

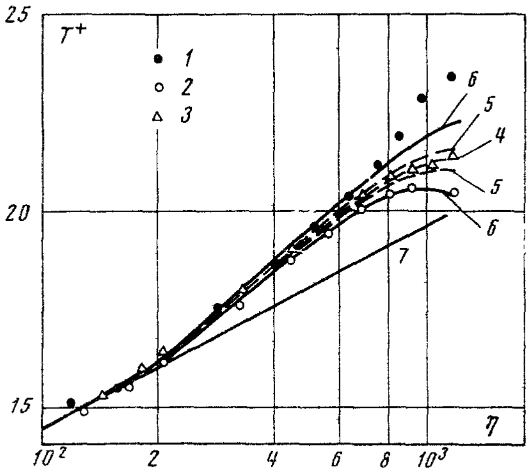

Fig. 2

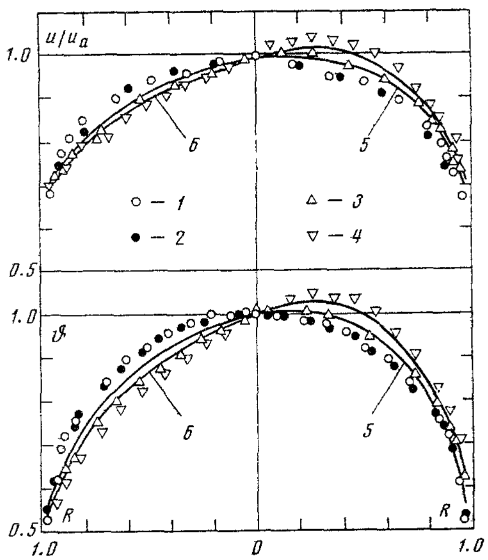

Fig. 3

We assume that

$$U_0^+ = 2.5 \ln \eta + 5.5, \quad \xi = 0.316 / \mathrm{Re}^{0.25}$$

for the velocity and frictional drag when thermogravitation is absent.

Equations (18) and (19) demonstrate that the deformation in the profile of velocity u and temperature profile appear, in turn, in the vertical center-line plane.

A calculation of the influence of thermogravitational forces solely in turbulent momentum transfer [4] yields expressions for the distribution of velocity and temperature near the upper and lower generatrices:

$$U^+ = U_0^+ \pm 3.8 \cdot 10^3 \frac{\mathrm{Gr}}{\mathrm{Re}^{3.5}\,\mathrm{Pr}}\,\eta \tag{20}$$

$$T^+ = T_0^+ \pm 3.8 \cdot 10^3 \frac{\mathrm{Gr}}{\mathrm{Re}^{3.5}\,\mathrm{Pr}}\,\eta \tag{21}$$

where the plus sign refers to distributions near the upper generatrix and the minus sign, near the lower generatrix.

It is evident from Eqs. (18)-(21) that the influence of thermogravitational forces on the velocity and temperature profiles for the two limiting cases (influence on turbulent transfer or on the averaged flow) differs. In both cases deformation of the profiles occurs in the same direction and begins at approximately the same values of the parameters.

In Fig. 2 the distribution of temperature in an air flow for $\mathrm{Re} = 5.2 \cdot 10^4$ and $\mathrm{Gr} = 10^9$ calculated using Eqs. (19) (curves 6) and Eq. (21) (curves 5) is compared to that found experimentally (points 1 and 2). Curve 4 for the temperature distribution in the absence of any influence of thermogravitation T_0^+ is constructed using experimental data (points 3), which refer to the horizontal center-line plane. It can be seen from Fig. 2 that the influence of thermogravitational forces in both cases begins to appear practically simultaneously. A calculation using Eq. (19) corresponds better to the experimental temperature profile. A substantial deviation of the experimental points from the calculated curve when $\varphi = 0$ is due to the simultaneous influence of thermogravitational forces on turbulent transfer and directly on the averaged flow. The influence on turbulent transfer in this case is substantial.

In Fig. 3 calculated distributions of velocity u/u_a and temperature $v = (t_W - t)/(t_W - t_a)$, where u_a and t_a are the values on the axes (curves 5 and 6) are compared to experimental data (points 3 and 4) in the vertical center-line plane in an air flow. The distributions in the horizontal center-line plane (points 1 and 2) found experimentally are taken as the distributions of velocity $(u/u_a)_0$ and of temperature v_0 in the absence of any influence of thermogravitation. The experimental points 1 and 3 and the calculated curves 5 refer to the values $\mathrm{Re} = 5.2 \cdot 10^4$ and $\mathrm{Gr} = 10^9$ and the experimental points 2 and 4 and calculated curves 6, to the values $\mathrm{Re} = 5.1 \cdot 10^4$ and $\mathrm{Gr} = 1.55 \cdot 10^9$. The variation in the Gr number under these conditions leads to substantial deformation of the profiles in the vertical center-line plane, the profiles remaining invariant in the horizontal center-line plane.

The distributions of u/u_a and ϑ in the horizontal center-line plane entirely coincide with that depicted in Fig. 3 for smaller Gr and identical Re. This corresponds to the results of a theoretical solution of Eqs. (18) and (19), which demonstrates that the profiles of the axial-velocity component u and temperature in the horizontal center-line plane are not deformed at the initial stage of influence of thermogravitational forces. The divergences between the calculated and experimental data in the upper part of the flow (left side of figure) is greater than in the lower side (right side of figure). This is due to our ne-

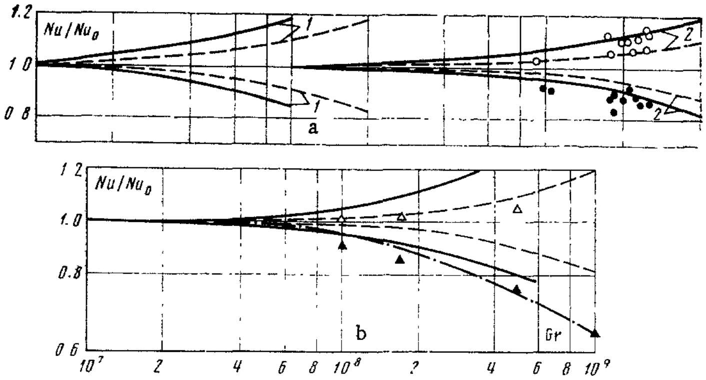

Fig. 4

glecting the influence of thermogravitational forces on turbulent transfer, which are more substantial near the upper generatrix than near the lower generatrix.

We calculate the variation in the local Nu number near the upper and lower generatrices from the equations

$$\overline{T}^{+} = \mathrm{Pe}_{*}/\mathrm{Nu} = T_a^{+} - T_{a0}^{+} + T_0^{+} \tag{22}$$

Using Eq. (19), we determine the Nu number at the initial stage of the development of secondary free-convection flow,

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_0} = 1 - \frac{340\mathrm{Gr}\cos\varphi}{\sqrt{\mathrm{Pr}}\,\mathrm{Re}^{2.75}}\left(1 + 2.4\frac{\mathrm{Pr}^{2/3} - 1}{\mathrm{Re}^{1/8}}\right)^{-1} \tag{23}$$
$$\mathrm{Nu}_0 = \frac{\mathrm{Re}\,\mathrm{Pr}\,\xi/8}{1.07 + 12.7\sqrt{\xi/8}\,(\mathrm{Pr}^{2/3} - 1)}$$

In Fig. 4 the variation of the relative Nusselt number $\mathrm{Nu}/\mathrm{Nu}_0$ at the upper and lower generatrices when $x/d > 40$ as a function of the Gr number for different values of Re and Pr is depicted. Experimental data previously [1] obtained for a water flow ($\mathrm{Pr} = 3.5$, $\mathrm{Re} = 1.2 \cdot 10^4$, curve b) and experimental data for an air flow ($\mathrm{Pr} = 0.7$, $\mathrm{Re} = 5.1 \cdot 10^4$, curve a) are depicted in the figure. Curves calculated for the corresponding experimental data of parameters using Eq. (23) (solid lines) and using dependences obtained in [4], which consider the influence of thermogravitation solely on turbulent transfer (dash–dot lines) are also entered in Fig. 4. The agreement between the results of the calculation for air and the experimental data appears satisfactory. The results of one of the calculations agrees better with the experimental data near the upper generatrix and the results of the second calculation, with the experimental data near the lower generatrix for the case of a water flow.

Such a difference in the nature of the variation of heat transfer may be due to the fact that the relations obtained for the initial stage of the process do not completely take into account the influence of the Pr number, which may otherwise appear at large Gr. Curves 1 in graph a of Fig. 4 correspond to $\mathrm{Re} = 1.2 \cdot 10^4$, as does graph b. The broken line in graph b was constructed using a previously obtained [1] empirical equation.

We may obtain from Eq. (23) a relation that determines the "one-percent" variation boundary of Nu on the upper (lower) generatrix as a result of the influence of thermogravitational forces. This relation has the form

$$\mathrm{Gr} = 3 \cdot 10^{-5}\sqrt{\mathrm{Pr}}\,\mathrm{Re}_2^{.75}\left[1 + 2.4(\mathrm{Pr}^{2/3} - 1)/\mathrm{Re}^{1/8}\right] \tag{24}$$

A comparison with experimental data obtained for a water flow and an air flow shows that Eq. (24) sharply demarcates the region of noticeable influence of thermogravitation on local heat transfer in horizontal tubes and the region of conditions lacking any thermogravitation influence.

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